High-Accuracy Assigned Power Excitation for the FDTD Technique

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***Abstract*— When the finite-difference time-domain (FDTD) technique is used in continuous wave excitation, the control of the effectively excited power is not allowed by classical approaches. In this letter two possible solutions, working also for nonuniform sampling of the computational domain, are proposed and illustrated. Examples of successfully applications for a metallic rectangular waveguide are, finally, reported.**

***Index Terms*—FDTD methods, Maxwell solver, waveguides.**

I. Introduction

THE finite-difference time-domain (FDTD) method [1] has found many applications in fields ranging from microwave telecommunication [2] and power applications [3] to optics [4]. FDTD characteristics are fully exploited in wide-frequency-range calculation of scattering parameters of microwave devices, where pulse excitations are required and problems like the absolute evaluation of the carried power can normally be neglected, because the scattering parameters depend on the ratios between field amplitudes. This is no longer true in power applications [3], where the control of the excited power becomes mandatory. The speed and the quality of the microwave treatment, in fact, depend on the electric field amplitude inside the product.

So far, the problem of exactly controlling the power of the input signal has not been considered in detail. In the following, we will show how to obtain two excitation schemes which allow impose the desired power. Examples will confirm their applicability even with irregular mesh discretizations.

II. Classical Excitation in FDTD Simulations

The source of the electromagnetic (EM) field for a FDTD simulation has always been implemented in two ways. The former, preferred for pulse excitations, replaces the calculated electric field $\mathbf{E}$ (boldface letters refer to vector fields) on the Yee-cell edge of the excitation section by the field generated by the source at each time step ("replaced source"). The latter adds the source $\mathbf{E}$ to the FDTD-calculated field ("added source") [5]. For a CW excitation the "added source" technique is preferred, as it does not alter the field resulting from reflections of waves propagating back to the source. This is typical in microwave heating, where steady-state EM distributions inside closed devices must be calculated. In these applications, the typical feeding system is a magnetron connected to a rectangular waveguide operating in single-mode regime. A FDTD code can easily simulate the field distribution far from the source simply adding in each cell of the input section the $\mathbf{E}$ and $\mathbf{H}$ transverse field components of TE_{10} mode. Assuming y and z as the transverse coordinates, the E_z field component in the Yee's scheme [1] can be written as shown in (1) at the bottom of the next page, being ε the dielectric permittivity and σ the conducibility in the cell, c_0 the speed of the light in vacuum and SF the *stability factor* (SF ≥ 1 to respect the Curant criterion [6]).

Because of the dependence of C_2 on the discretization parameters (via Δt), the amplitude of the EM field calculated by the FDTD algorithm depends on both the spatial and temporal grids and the SF factor. This cannot be accepted when simulating power application devices, where the excited power must be independent from the computational parameters. To highlight this problem, let us consider the TE_{10} mode propagating in a 50-cm-long rectangular R26 waveguide at the frequency of 2.45 GHz. Without loss of generality, this problem can be reduced to a two-dimensional (2-D) one. We want the mode power be 1 kW, corresponding to amplitudes of the transverse electric and magnetic field components $E_z = 24.67 \times 10^3$ V/m and $H_y = 44.88$ A/m, respectively [7]. As a reference simulation (R) uniform steps $\Delta x = \Delta y = 2$ mm and stability factor SF $= 1$ (corresponding to a maximum allowed $\Delta t = 4.71$ ps) are assumed. To test all the possible combinations of simulation parameters other input data sets have been considered: $\Delta x = \Delta y = 2$ mm, SF $= 2$, $\Delta t = 2.36$ ps (label A); $\Delta x = 1$ mm (x from 0–25 cm), $\Delta x = 2$ mm (elsewhere), $\Delta y = 2$ mm, SF $= 1$, $\Delta t = 2.98$ ps (label B); $\Delta x = 2$ mm (x from 0 to 25 cm), $\Delta x = 1$ mm (elsewhere), $\Delta y = 2$ mm, SF $= 1$, $\Delta t = 2.98$ ps (label C); $\Delta x = 2$ mm, $\Delta y = 1$ mm, SF $= 1$, $\Delta t = 2.98$ ps (label D).

Simply adding the exciting electric field to the E_z Yee's equations, all the performed simulations lead to results evidencing the expected dependance of the fields amplitude, as suggested by the expression of $E_{\mathrm{INC},z}$ shown in (2) at the bottom of the next page, and incorrect values of the calculated field components, quite different from their expected maximum values. Also using both E_z and H_y components for the excitation, as it should be for a TE_{10} mode, results do not change substantially.

In the next section two different methods to overcome this problem will be presented and their ability to excite an assigned power will be illustrated.

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III. The "Fixed-Power" Excitation Schemes

The two techniques we propose to set the mode power require the knowledge of the transverse components of the exciting field and can be applied to both TE and TM cases. In the following, for simplicity, attention will be restricted to only the TE_{10} mode excitation.

In the first proposed Electro Magnetic Excitation Scheme (EMES-1), the computation domain is divided in two regions: the "*excitation section*," where the sources are placed, and the remaining volume, identified as the "*propagation domain*." In the latter, the *total field* formulation of the FDTD method is used. In the former, the field is calculated using an approach formally similar to the *scattered field formulation* [8] of the FDTD method, considering the *total field* in this excitation plane as a combination of the *scattered field* coming from the "*propagation domain*" and the *incident field*, produced by the source and analytically known. This allows to control the absolute excited power and also the unmodified propagation of any back-scattered field through the excitation plane.

In our formulation, the equations for both the *total* (suffix t) and *scattered* (suffix s) field components can be written as shown in (3) at the bottom of the page, where D_1 and D_2 correspond to C_1 and C_2, but are evaluated using the dielectric properties μ and τ instead of ε and σ. The distinction between *scattered* and *total* field holds only for the excited field components, E_z and H_y in this case. For all the others, the regular Yee's expression must be used.

The second scheme (EMES-2) is even simpler. The EM field is computed in the whole domain following the *total field* formulation of the "*propagation domain*," as defined for the EMES-1. This scheme comes from the "total/scattered" field formulation of the FDTD method [6], modified for a metallic rectangular waveguide excitation. In the excitation plane, the source-generated $E_{\mathrm{INC},z}$ field is now added to the E_z field before its calculation, via Yee's equations, in the time step $n+1$ when the source $H_{\mathrm{INC},y}$ field is subtracted to the previously calculated H_y field.

This procedure annihilates the contribution of the *incident* E_z field to the *total* H_y field on one side of the source plane, resulting in a one-way propagation that depends on the chosen annihilation plane. Any back-scattered field can still go through the excitation plane as it contributes to the *scattering* part of the *total* field computation and is not modified by this excitation procedure, which involves only the *incident* field. The Yee's equations for both the transverse components in the excitation plane must be modified as shown in (4) at the top of the next page.

In practice, the choice between the two schemes can be done observing that the former excites the propagation of the EM wave in both directions whereas the latter launches the field in only one.

Using the two proposed excitation schemes, whichever technique and set of data is used, the temporal field evolution always superimpose and the fields exhibit the expected values: $E_z = 24.67 \times 10^3$ V/m and $H_y = 44.88$ A/m.

To analyze more quantitatively the accuracy of these two EMES's for all the parameter sets, the flux of the active power in a section of the rectangular waveguide has been

$$E_z^{n+1}(i,\, j,\, k+\tfrac{1}{2}) = C_1 E_z^n\big(i,\, j,\, k+\tfrac{1}{2}\big) + C_2(\nabla \times H)_z + E_{\mathrm{INC},\,z}^{n+1}(i,\, j,\, k+\tfrac{1}{2}) \tag{1}$$

$$E_{\mathrm{INC},\,z}^{n+1}(i,\, j,\, k+\tfrac{1}{2}) = C_2 J_s(i,\, j,\, k+\tfrac{1}{2}) \qquad \Delta t \le \frac{1}{\mathrm{SF}\, c_0\sqrt{1/(\Delta x_{\min})^2 + 1/(\Delta y_{\min})^2 + 1/(\Delta z_{\min})^2}}$$

$$C_1 = \left[\frac{1}{\Delta t} - \frac{\sigma}{2\varepsilon}\right] \Bigg/ \left[\frac{1}{\Delta t} + \frac{\sigma}{2\varepsilon}\right]_{(i,\, j,\, k+1/2)}, \qquad C_2 = \left[\varepsilon\left(\frac{1}{\Delta t} + \frac{\sigma}{2\varepsilon}\right)\right]^{-1}_{(i,\, j,\, k+1/2)} \tag{2}$$

$$\begin{aligned}
E_{t,\,z}^{n+1}\left(i,\, j,\, k+\frac{1}{2}\right) &= C_1 E_{t,\,z}^{n}\left(i,\, j,\, k+\frac{1}{2}\right) + C_2\left[\frac{H_x^{n+1/2}(i,\, j-\frac{1}{2},\, k+\frac{1}{2}) - H_x^{n+1/2}(i,\, j+\frac{1}{2},\, k+\frac{1}{2})}{\Delta y}\right] \\
&\quad + C_2\left\{\frac{[H_{s,\,y}^{n+1/2}(i+\frac{1}{2},\, j,\, k+\frac{1}{2}) + H_{\mathrm{INC},\,y}^{n+1/2}(i+\frac{1}{2},\, j,\, k+\frac{1}{2})] - H_{t,\,y}^{n+1/2}(i-\frac{1}{2},\, j,\, k+\frac{1}{2})}{\Delta x}\right\} \\
E_{s,\,z}^{n+1}\left(i,\, j,\, k+\frac{1}{2}\right) &= C_1 E_{s,\,z}^{n}(i,\, j,\, k+\tfrac{1}{2}) + C_2\left[\frac{H_{s,\,y}^{n+1/2}(i+\frac{1}{2},\, j,\, k+\frac{1}{2}) - H_{s,\,y}^{n+1/2}(i-\frac{1}{2},\, j,\, k+\frac{1}{2})}{\Delta x}\right] \\
H_{s,\,y}^{n+1/2}\left(i+\frac{1}{2},\, j,\, k+\frac{1}{2}\right) &= D_1 H_{s,\,y}^{n+1/2}\left(i+\frac{1}{2},\, j,\, k+\frac{1}{2}\right) + D_2\left[\frac{E_x^n(i+\frac{1}{2},\, j,\, k) - E_x^n(i+\frac{1}{2},\, j,\, k+1)}{\Delta z}\right] \\
&\quad + D_2\left\{\frac{E_{s,\,z}^n(i+1,\, j,\, k+\frac{1}{2}) - [E_{t,\,z}^n(i,\, j,\, k+\frac{1}{2}) - E_{\mathrm{INC},\,2}^n(i,\, j,\, k+\frac{1}{2})]}{\Delta x}\right\}
\end{aligned} \tag{3}$$

$$
\begin{aligned}
H_y^{n+1/2}\left(i+\frac{1}{2}, j, k+\frac{1}{2}\right) &= D_1 H_y^{n+1/2}\left(i+\frac{1}{2}, j, k+\frac{1}{2}\right) + D_2\left[\frac{E_x^n(i+\frac{1}{2}, j, k) - E_x^n(i+\frac{1}{2}, j, k+1)}{\Delta z}\right] \\
&\quad + D_2\left\{\frac{E_z^n(i+1, j, k+\frac{1}{2}) - \left[E_z^n(i, j, k+\frac{1}{2}) + E_{\mathrm{INC},z}^n(i, j, k+\frac{1}{2})\right]}{\Delta x}\right\} \\
E_z^{n+1}\left(i, j, k+\frac{1}{2}\right) &= C_1 E_z^n\left(i, j, k+\frac{1}{2}\right) + C_2\left[\frac{H_x^{n+1/2}(i, j-\frac{1}{2}, k+\frac{1}{2}) - H_x^{n+1/2}(i, j+\frac{1}{2}, k+\frac{1}{2})}{\Delta y}\right] \\
&\quad + C_2\left\{\frac{\left[H_y^{n+1/2}(i+\frac{1}{2}, j, k+\frac{1}{2}) - H_{\mathrm{INC},y}^{n+1/2}(i+\frac{1}{2}, j, k+\frac{1}{2})\right] - H_y^{n+1/2}(i-\frac{1}{2}, j, k+\frac{1}{2})}{\Delta x}\right\}
\end{aligned}
\tag{4}
$$

TABLE I
ACTIVE POWER IN A SECTION OF THE RECTANGULAR WAVEGUIDE FOR THE FIVE DIFFERENT SIMULATIONS AND RELATIVE MEASURED ERROR AFTER 15 000 TIME STEPS

Simulation	EMES-1		EMES-2	
	Power (W)	Err (%)	Power (W)	Err (%)
R	1022.70	2.27	1011.66	1.17
A	1021.80	2.18	1011.27	1.13
B	1019.91	1.99	1011.48	1.15
C	1022.11	2.21	1010.73	1.07
D	1022.94	2.29	1012.53	1.25

TABLE II
ACTIVE POWER IN A SECTION OF THE RECTANGULAR WAVEGUIDE AND RELATIVE MEASURED ERROR USING A HANNING WINDOW WITH A TIME CHARACTERISTIC OF TEN SINUSOIDAL PERIODS AFTER 15 000 TIME STEPS

Simulation	EMES-1		EMES-2	
	Power (W)	Err (%)	Power (W)	Err (%)
R	1000.60	$6.0\ 10^{-2}$	1000.05	$5.0\ 10^{-3}$
A	1000.67	$6.7\ 10^{-2}$	1000.20	$2.0\ 10^{-2}$
B	999.89	$1.15\ 10^{-2}$	1000.44	$4.4\ 10^{-2}$
C	1000.98	$9.8\ 10^{-2}$	1000.39	$3.9\ 10^{-2}$
D	1001.83	$1.83\ 10^{-1}$	1001.47	$1.47\ 10^{-1}$

evaluated. The results obtained after 15 000 time steps (when a steady-state condition can be reasonably supposed) for both the excitation methods are reported in Table I. The error is always lower than 2.3%, much smaller than the experimental inaccuracy related to the practical microwave excitation devices. Results obtained after 150 000 time steps are even better (errors about two orders of magnitude lower), because all of the excited spurious harmonic contributions excited when the source is turned on have been eliminated.

To obtain a good precision in the power excitation procedure also with fewer time step number, both proposed excitation schemes can be integrated with the method illustrated in [9], which only affects their temporal evolution modulating the sinusoidal behavior with a Hanning function. Results reported in Table II, relative to the 15 000 time step simulations, show errors reduced by more than one order of magnitude.

IV. CONCLUSIONS

Two excitation schemes, both based on some modifications of the classical FDTD formulations, have been proposed to fix exactly the power of an exciting CW source as needed in FDTD simulations of microwave power applicators. The performances of both the procedures, valid for TE and TM excitation modes and 2-D or 3-D simulation cases, have been tested successfully for the TE_{10} mode of a rectangular waveguide with different sets of parameters.

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